In this unit you will learn to:

- recognize an angle and its properties.
- estimate and measure angles using a protractor and express their measurements in degrees.
- recognize regular and irregular polygons.
- classify triangles according to their angle measurements and side lengths.
- recognize how to measure the angles of triangles and quadrilaterals.
- construct triangles using geometric tools based on their side lengths and angle measurements.
- show an organized and neat work style.

Key Words

segment
vertex
acute
obtuse
estimate
regular polygon

The Eiffel Tower in Paris, France, is 324 m tall. It was built in 1889 and was the tallest structure in the world for 41 years. It is one of the most famous monuments in the world.

The Eiffel Tower is formed by triangles. Triangles are the only polygons that do not change when force is applied to them.
What Do You Know?

Look at the images and answer.

1. Mark the angle with the larger opening with a "✓".
   a. [Image of angle with "✓"]
   b. [Image of angle without "✓"]

2. Write the name of the geometric figure shown in each image.
   a. [Image of geometric figure]
   b. [Image of geometric figure]

3. Draw two polygons that are different from those above.

   [Blank area for drawing]
Angles and Their Properties

Connecting

A car’s windshield wiper cleans the windshield by moving from one side to another.

• In the image each letter represents the following:
  ‣ the fixed point of the windshield wiper is \( O \).
  ‣ segment \( OA \) connects the fixed point, \( O \), to the windshield wiper in its normal position, \( A \).
  ‣ segment \( OB \) connects the fixed point, \( O \), to the farthest position of the windshield wiper, \( B \).

• Mark the geometric shape represented in the image.

Learning

An angle (\( \angle \)) is the space between, or shape formed by two lines extending from the same point. The lines are called segments, and the point where the two lines meet is called the vertex.

Generally, an angle is labeled with capital letters and is read counterclockwise. The middle letter indicates its vertex.

Examples:

- Angle \( ABC (\angle ABC) \)
- Angle \( DEF (\angle DEF) \)

Sides: \( BA \) and \( BC \)  
Vertex: \( B \)

Sides: \( ED \) and \( EF \)  
Vertex: \( E \)
1. Complete with information from the angles. Identify

   a. Angle $\angle PAL$
   
   Sides: ____________________
   Vertex: ____________________

   b. Angle $\angle DIL$
   
   Sides: ____________________
   Vertex: ____________________

   c. Angle $\angle FMP$
   
   Sides: ____________________
   Vertex: ____________________

2. Label each angle. Identify

   a. $\angle ABC$
   
   ____________________

   b. $\angle RPQ$
   
   ____________________

   c. $\angle MJQ$
   
   ____________________

3. Read the situation and answer. Analyze

   A door opens and forms an angle. The segments that correspond to the initial and final position can be drawn in the following way:

   a. Which letter of the angle is the vertex?

   ____________________

   b. How do you write the sides of the angle?

   ____________________
Measuring Angles

Connecting

Matt and Valerie are looking at two buildings from their bases to their highest points. Valerie is looking at the taller building. Matt is looking at the shorter one. They are standing the same distance from the buildings.

- Mark each correct statement with a ✓.
  - Matt is looking up higher than Valerie.
  - Valerie is looking up higher than Matt.
  - Matt and Valerie are looking up at the same inclination.

Learning

To **measure** an angle, you can use a **protractor**.

An **angle** is measured **counterclockwise**, following these steps:

- place the small hole, or origin, of the protractor over the vertex of the angle, and line up one of the sides with 0°.

- identify the measurement on the protractor that coincides with the other side of the angle.

**Example:** To measure Angle ABC, you have to line up the protractor like so:

Therefore, the measurement for Angle ABC is 70°.
1. Use a protractor to measure each angle. **Apply**

   ![Diagram](image)

   **a.** Measurement = __________
   **c.** Measurement = __________

   ![Diagram](image)

   **b.** Measurement = __________
   **d.** Measurement = __________

2. Read the situation and answer. **Analyze**

   Andrea and Michael are looking at the angles of their staplers.

   - The angles are the same.
   - The blue stapler’s angle is greater.

   • Who is correct? Explain.
Classifying Angles

Connecting

Archery is a sport that consists of shooting an arrow at the center of an object called a target. The goal is to get the most points by shooting closest to the center.

• Look at the point where the bowstring meets the arrow and circle the correct option.
  ▶ An angle between 0° and 90° is formed.
  ▶ Two angles between 0° and 90° are formed.
  ▶ Three angles between 0° and 90° are formed.

• Mark the statement that shows what happens when the bowstring is stretched to shoot the arrow.
  [ ] It forms an angle between 0° and 90°.  [ ] It forms an angle between 90° and 180°.

Learning

Based on their measurements, angles are classified as:

- **Acute**: greater than 0° but less than 90°
- **Right**: 90°
- **Obtuse**: greater than 90° but less than 180°
- **Straight**: 180°
- **Complete**: 360°
Practicing

1. Classify each angle according to its measurement. Classify

   a. Classification: ____________
   b. Classification: ____________
   c. Classification: ____________
   d. Classification: ____________

2. Use a protractor to measure the angles. Then classify them. Analyze

<table>
<thead>
<tr>
<th>Angle</th>
<th>Measurement</th>
<th>Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>∠ ESD</td>
<td></td>
<td></td>
</tr>
<tr>
<td>∠ GLK</td>
<td></td>
<td></td>
</tr>
<tr>
<td>∠ YLS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>∠ JYL</td>
<td></td>
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</tr>
</tbody>
</table>

3. Identify the error in the statement and correct it. Verify

   The sum of two acute angles is always less than a right angle.

   Error: _____________________________________________
   Correction: _________________________________________

Angles and Polygons
Estimating Angles

Connecting

A neighborhood's houses have roofs with different angles.

- Mark the sentence that is correct with a ✓.
  - The blue house has a roof with an angle greater than 90°.
  - The red house has a roof with an angle greater than 90°.
  - The yellow house has a roof with an angle less than 90°.

- Which house has a roof that is a right angle? Trace the correct answer.
  - red house
  - blue house
  - green house
  - yellow house

Learning

When we do not know an angle's exact measurement, we can estimate it. We can estimate by using our previous knowledge and by thinking logically.

Example:

- We can estimate that Angle $DRC$ is greater than 90° and less than 180° and is approximately 170°. We can also determine that the angle is obtuse.
- Using this section of the London Underground map, we can estimate that the angles where the lines intersect measure 90° and, therefore, are right angles.
1. Classify the angles without measuring them. Then estimate their measurements. Look at the example. Recognize

Angle $ABC$ is classified as an acute angle. We can estimate that it is between $50^\circ$ and $60^\circ$. Therefore, it should measure approximately $55^\circ$.

Quiz Yourself

- Using a protractor, measure the angles in the red circles on the London Underground map.
- In a different color, circle the obtuse angles on the Piccadilly Line (the dark blue line).
- Estimate the measurement of the yellow angle. Then use your protractor to measure the angle and check your estimate.
Let's Check!

Angles and Their Properties

1. Complete the information.
   a. b. c.
   
   Vertex: ____________
   Sides: ____________
   
   Vertex: ____________
   Sides: ____________
   
   Vertex: ____________
   Sides: ____________

Measuring Angles

2. Use a protractor to measure each angle.
   a. b. c.
   
   Measurement = ______
   Measurement = ______
   Measurement = ______

Classifying Angles

3. Classify the angles as acute, obtuse or straight.
   a. b. c.
   
   Classification: ____________
   Classification: ____________
   Classification: ____________
Estimating Angles

4. Analyze the figure and answer.
   a. Write the names of 5 obtuse angles.
   
   ____________________________
   b. What is the approximate measurement of Angle DGA?
   
   ____________________________
   c. What is the sum of Angles NGF’s and AGN’s measurements?
   
   ____________________________

Using Geometry Tools

5. Draw angles using the measurements.
   a. $\angle BEA$ is 50°.

   ____________________________
   b. $\angle FGH$ is 165°.

   ____________________________

6. Draw the angle.
   The angle equals 180° when added to $\angle FSZ$.

   ____________________________
Irregular and Regular Polygons

Connecting

A group of scouts went camping. They set up their tents, marking the borders with stakes. This is what it looked like from above.

• Circle the correct statement.
  All of the sides of Figure 2 are the same length.  All of the sides of Figure 4 are different lengths.

• Mark the correct statement with a ✔️ and the incorrect statement with an ❌.
  ✔️ Figure 1 has six interior angles.  ❌ Figure 3 has four interior angles.

Learning

A **polygon** is a closed geometric figure that has three or more sides that are all joined at the vertices. A **regular polygon** is a figure whose interior angles and sides are equal or congruent. If a polygon is incongruent, it is an **irregular polygon**.

**Examples:** Below you can see a regular and an irregular octagon.

- Regular octagon
- Irregular octagon
1. Color the regular polygons red and the irregular polygons blue. Classify

![Regular and Irregular Polygons Diagram]

2. Write regular or irregular, according to the characteristics. Analyze

![Quadrilateral Diagrams]

2. Write regular or irregular, according to the characteristics. Analyze

![Quadrilateral Diagrams]

3. Write T for true and F for false. Explain. Evaluate

a. Trapezoids are regular polygons.
   Explain: 

b. Trapeziums are irregular polygons.
   Explain: 

c. All parallelograms are regular polygons.
   Explain: 

Remember!

Quadrilaterals can be:
- Parallelograms: have all opposite sides that are parallel.
- Trapezoids: have one pair of parallel sides.
- Trapeziums: have no parallel sides.
Triangles

Connecting

A volleyball coach told his team to stand in this formation on the court.

- If you connect points A, B and C, what figure is formed? ▶

- Write T for true and F for false.
  - Points D, E and F form a triangle.
  - The sum of the lengths of segments DE and BD is greater than the distance between B and E.

Challenge

Research what a “truss” is. Then identify three structures where you might find a truss.

Learning

A triangle is a polygon with three sides, three vertices and three interior angles.

According to the lengths of their sides, triangles can be classified as:

- **Equilateral**: All sides are equal.
- **Isosceles**: Two sides are equal.
- **Scalene**: All sides are different.

All triangles fit the rule of triangle inequality, meaning that, for a triangle to exist, each side must measure less than the sum of the other two.

\[
\begin{align*}
  a + b &> c \\
  b + c &> a \\
  a + c &> b
\end{align*}
\]
1. Classify the triangles according to the lengths of their sides.

   a. ![Triangle with sides 6 cm, 10 cm, 8 cm]
   b. ![Triangle with sides 3.2 mm, 3.2 mm, 4.8 mm]
   c. ![Triangle with sides 5 cm, 5 cm, 5 cm]
   d. ![Triangle with sides 6 m, 6 m, 1.5 m]
   e. ![Triangle with sides 8 cm, 5 cm, 6 cm]
   f. ![Triangle with sides 3 cm, 3 cm, 3 cm]

2. Explain whether or not the measurements correspond to the sides of a triangle.

   a. 4 cm, 6 cm and 10 cm
   b. 30 mm, 30 mm and 50 mm
   c. 2 dm, 2 dm and 6 dm

3. Write T for true and F for false. Explain.

   a. □ The sides of a scalene triangle do not fit the rule of triangle inequality.
      Explain: 
      
   b. □ The equal sides of an isosceles triangle are less than the length of its third side.
      Explain: 
      
   c. □ If a triangle is equilateral, the sum of the lengths of two of its sides is double the length of the third.
      Explain: 
      
   d. □ There is a scalene triangle whose sides are 5 cm, 4 cm and 9 cm.
      Explain: 

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Angles and Polygons
Angles of a Triangle

Connecting

Annie knows how to find out what the sum of the interior angles of a triangle is.

- What is the measurement of a straight angle? [ ]
- Cut a piece of paper to form a triangle. Then fold each of the vertices so that they meet with the opposite side.
- What type of angle is \( \angle CBA \)? [ ]
- What is the sum of the interior angles of the triangle? Explain.

Learning

**Triangles** can be classified by the measurement of their **interior angles**, which always add up to 180°.

**Acute Triangle**

- All of its interior angles are **acute**, meaning they are less than 90°.

**Right Triangle**

- It has **one interior right angle**, meaning it is a 90° angle.

**Obtuse Triangle**

- It has **one interior obtuse angle**, meaning one angle is more than 90° but less than 180°.
1. Classify the triangles according to their interior angles. Classify

   a. 
   
   b. 
   
   c. 

2. Write T for true and F for false. Explain. Evaluate

   a. □ A right triangle has one right angle and one obtuse angle.
      Explain: 

   b. □ All equilateral triangles are acute.
      Explain: 

   c. □ If an isosceles triangle has one obtuse angle, the other two are acute.
      Explain: 

   d. □ An obtuse triangle has three obtuse angles.
      Explain: 

3. Calculate the interior angles of the triangles. Calculate

   a. 
   
   b. 

Did You Know...?

Two angles of an isosceles triangle are always the same.
Angles of a Quadrilateral

Connecting

To prove that the sum of the interior angles of a quadrilateral is 360°, Ryan does the following:

- He cuts a piece of paper in the shape of a quadrilateral. Tracing the segment AC, he forms two triangles, as shown below.

![Diagram of a quadrilateral cut into two triangles]

- Mark the correct statement with a ☑.

  - The sum of the interior angles of a triangle is always 180°.
  - The sum of the interior angles of a triangle is always 120°.

- What is the sum of the interior angles of a quadrilateral? Explain.

   ____________________________________________________________________________

Learning

**Quadrilaterals** are polygons made up of four straight sides. The sum of the **interior angles** of a quadrilateral is always 360°.

**Example:** What is the measurement of $\angle FED$ in the quadrilateral?

The sum of the interior angles must equal 360°:

$70^\circ + 120^\circ + 73^\circ + ? = 360^\circ$

then

$360^\circ - 70^\circ - 120^\circ - 73^\circ = ?$

$= 97^\circ$
1. Calculate the missing angles of the quadrilaterals. Apply

a. 

b. 

c. 

d. 

2. Write T for true and F for false. Explain. Evaluate

a.  

A square’s angles are all equal.

Explain: 

b.  

A rectangle has interior angles with different measurements.

Explain: 

Quiz Yourself

Inside Hexagon ABCDEF there are two trapezoids.

- What is the measurement of the missing angle?

- What is the sum of the interior angles of Hexagon ABCDEF? Explain.
Connecting

A triangle can be drawn using a ruler and a compass if you know the lengths of its sides.

- The following segments are 3, 4 and 5 centimeters long.

- Draw the longest side with a ruler. Then, from each end of the line, draw arcs as long as the lengths of the segments, using a compass, and write down the lengths of the sides.

- Considering the lengths of its sides, write what type of triangle it is.  

Learning

By knowing the **lengths of all the sides of a triangle** it is possible to draw a triangle using a geometric tool called a compass. You can draw a triangle if you know the lengths of sides \( a \), \( b \) and \( c \) following the steps below.

1st Draw a line for side \( a \).

2nd From one end of side \( a \), use a compass to draw an arc with the measurement of side \( c \). You can use a ruler to measure the width of the compass.

3rd From the other end of side \( a \), use the compass to, draw an arc with the measurement for side \( b \).

4th Draw straight lines from each end of side \( a \) to the point where the arcs intersect.
1. Use the line segments to create triangles. Then answer. **Apply**

**a.**

3 cm

5 cm

3 cm

b.

4 cm

4 cm

4 cm

**c.** Use a protractor to measure the interior angles of each triangle. Then compare them and write down the differences.

---

2. Using geometric tools, draw an exact copy of the triangle. Then mark its sides and angles. **Analyze**

![Diagram of triangle with sides labeled 4 cm, 4 cm, and 3 cm.](image-url)
If you know the angles of a triangle, you can make many triangles with the same shape, but in different sizes. If two triangles have the same interior angles, they will always be the same shape, but they may not always be the same size.

Example: The red triangle has the same interior angles as the purple triangle, but the lengths of their sides are different.

Connecting
To draw a triangle with interior angles of 60°, 40° and 80°, you can do the following:

• Draw the baseline segment $CA$, 5 cm long, and make the angle at vertex $C$ measure 40°. Then make the angle at vertex $A$ measure 60° and draw a segment that intersects with $CB$. The other angle will be 80°.

• Is it possible to make another triangle with the same angles but with different side lengths? Explain.

Learning
If you know the angles of a triangle, you can make many triangles with the same shape, but in different sizes. If two triangles have the same interior angles, they will always be the same shape, but they may not always be the same size.

Example: The red triangle has the same interior angles as the purple triangle, but the lengths of their sides are different.
1. Draw the triangles. Apply
   a. A triangle with angles that measure 40°, 70° and 70°.
   
   b. A triangle with angles that measure 40°, 110° and 30°.

2. Draw triangles with the following characteristics. Analyze
   a. One angle measures 50° and another one is 60°. The length of one side is 5 cm.
   b. One angle measures 90°. The length of one side is 3 cm and another measures 4 cm.
Let’s Check!

Regular and Irregular Polygons
1. Measure the angles and sides of the polygons. Then classify them as regular or irregular.
   a.  
   b.  
   c.  

   [Polygons drawn]

   ▶️ ▶️ ▶️

Triangles
2. Measure the triangles and classify them based on the length of their sides.
   a.  
   b.  
   c.  

   ▶️ ▶️ ▶️

Angles of a Triangle
3. Look at the triangle and complete with the missing angles.

   [Triangle with angles labeled]

   ▶️ ▶️ ▶️
Angles of a Quadrilateral

4. Write the missing angles of the quadrilaterals.

a. 

b. 

Drawing a Triangle According to Its Side Lengths

5. Draw a triangle with the side lengths of 2 cm, 4 cm and 5 cm.

Drawing a Triangle According to Its Angles

6. Draw a triangle with one side length of 4 cm and two interior angles of 25° and 80°.
Multiple Choice Questions

1. This figure is formed of a regular pentagon and a square. What is the measurement of Angle $GAE$?

A. $18^\circ$
B. $72^\circ$
C. $108^\circ$
D. $198^\circ$

Analyzing the Answers

A. Angle $EDH$ measures $180^\circ$. So if we subtract $CDH (72^\circ)$ from $180^\circ$ we can calculate that $CDE$ measures $108^\circ$. The pentagon is regular, so all of its interior angles are equal, meaning $BAE$ is $108^\circ$. The interior angles of a square all measure $90^\circ$. So, $GAE$ must be $108^\circ - 90^\circ = 18^\circ$.

B. In this option, Angle $GAE$ is calculated as the Angle $EAI$, but this measurement coincides with Angle $CDH$.

C. In this option, $180^\circ - 72^\circ$ is calculated. This gives you the interior angle of the pentagon, but it does not finish the calculations needed to obtain the angle.

D. In this option, the interior angle of the pentagon and the interior angle of the square are added: $108^\circ + 90^\circ = 198^\circ$, a value that does not correspond to the Angle $GAE$.

Therefore, A is the correct answer.
What Did You Learn?

1. Identify 9 different angles in the figure. Then write their names.

2. Use a protractor to measure the angles. Then classify them.

<table>
<thead>
<tr>
<th>Angle</th>
<th>Measurement</th>
<th>Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>∠UPQ</td>
<td></td>
<td></td>
</tr>
<tr>
<td>∠VXQ</td>
<td></td>
<td></td>
</tr>
<tr>
<td>∠RUP</td>
<td></td>
<td></td>
</tr>
<tr>
<td>∠PUZ</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3. Classify the polygons as regular or irregular.

\[ \text{a.} \quad \begin{array}{c}
2 \text{ cm} & 113^\circ & 2 \text{ cm} \\
67^\circ & 67^\circ \\
2 \text{ cm} & 113^\circ & 2 \text{ cm}
\end{array} \]

\[ \text{c.} \quad \begin{array}{c}
60^\circ & 60^\circ & 60^\circ \\
60^\circ
\end{array} \]

4. Write the missing angles.

\[ \text{a.} \quad \begin{array}{c}
\_ \_ \_ \_ \_
\end{array} \]

\[ \text{b.} \quad \begin{array}{c}
115^\circ
\end{array} \]

5. Using a ruler and a compass, verify whether or not the triangles can be made. Then circle the correct answer.

\[ \text{a.} \quad \text{An equilateral triangle with one side that is 3 cm long.} \]

\[ \text{b.} \quad \text{An isosceles triangle with side lengths of 2 cm, 2 cm and 5 cm.} \]
6. Use the segments and angles to draw triangles.

   a. 
   
   b. 

7. To measure an angle with a protractor we must measure it:

   A. Clockwise
   B. Counterclockwise
   C. In any direction.
   D. In the direction of the second hand of a clock.

8. How would you classify angles whose measurements are 90°, 180°, 32°, 123° and 360°?

   A. Acute, obtuse, acute, obtuse and full.
   B. Right-angle, obtuse, acute, obtuse and full.
   C. Right-angle, straight, acute, obtuse and full.
   D. Right-angle, straight, acute, obtuse and straight.
Mark the correct answer with an $\times$.

9. Which of the polygons is regular?
   A. Trapezium
   B. Rectangle
   C. Square
   D. Isosceles triangle

10. How can you classify the triangle?
   A. Equilateral triangle
   B. Right isosceles triangle
   C. Acute scalene triangle
   D. Acute isosceles triangle

11. The following are angles of different triangles. Which groups represents a right scalene triangle?
   A. $50^\circ$, $60^\circ$ and $70^\circ$
   B. $30^\circ$, $60^\circ$ and $90^\circ$
   C. $45^\circ$, $45^\circ$ and $90^\circ$
   D. $20^\circ$, $40^\circ$ and $120^\circ$

12. This figure is a regular hexagon. What is the measurement of the interior angles?
   A. $60^\circ$
   B. $90^\circ$
   C. $120^\circ$
   D. $180^\circ$
13. What is the interior angle of each vertex in an equilateral triangle?

A. 30°
B. 60°
C. 120°
D. 180°

14. What is the value of the sum of the interior angles of the triangle?

A. 100°
B. 130°
C. 150°
D. 180°

15. Which description corresponds to an acute angle?

A. It measures more than 90°.
B. It equals 180°.
C. It measures more than 0° and less than 90°.
D. It measures more than 90° and less than 180°.

16. Which describes a right angle?

A. The angle between a wall and a part-opened door.
B. The angle in each corner of a square-shaped table.
C. The angle between the floor and a ladder leaning on a wall.
D. The angle between the center of an 8-slice cake and the two sides of one of the slices.